Formal Analysis of the Entropy / Security Trade-off in First-Order Masking Countermeasures against Side-Channel Attacks

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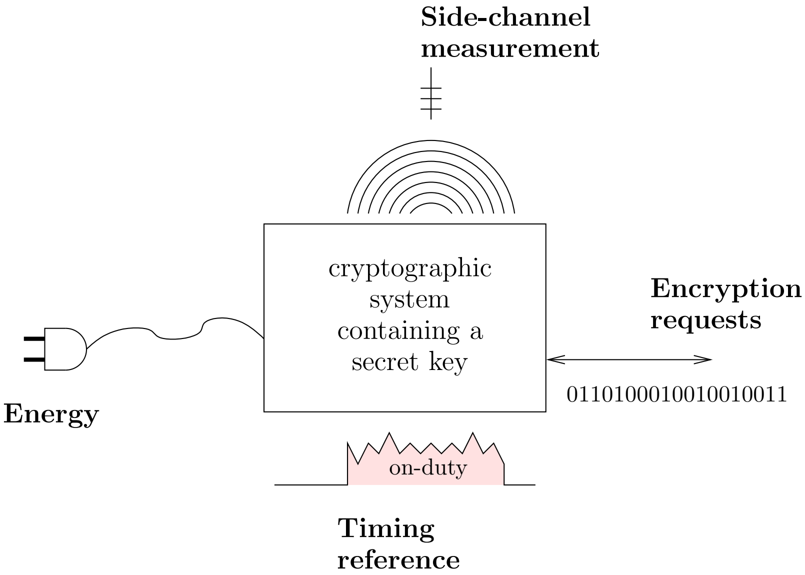
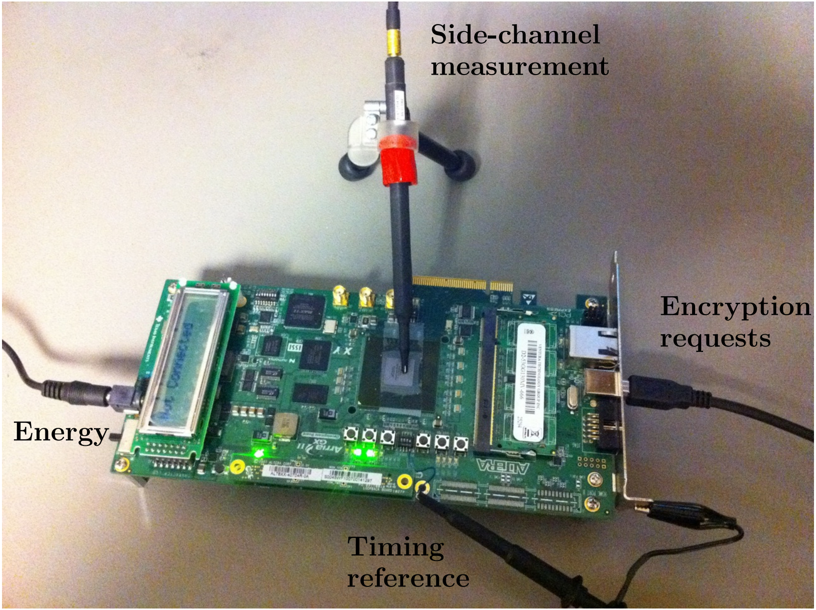
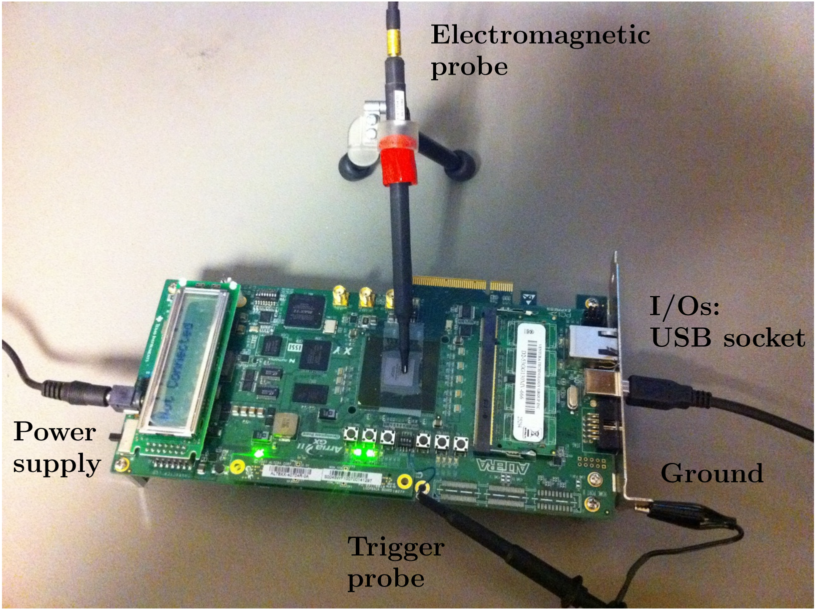

Side-channel measurement
cryptographic system containing a secret key
Encryption requests
0110100010010010011
Energy
on-duty
Timing reference


Side-channel measurement
Encryption requests
Energy
Timing reference


Electromagnetic probe
I/Os: USB socket
Power supply
Ground
Trigger probe

Protection against side-channel attacks

Extrinsic countermeasures

- Noise addition . . makes the attack difficult but not impossible
- Internal powering . . . . can be tampered with

Internal countermeasures

- Make the power constant . . require design skills [DGBN09] ✖
- Masking the power . . . . susceptible to HO-SCA ✔

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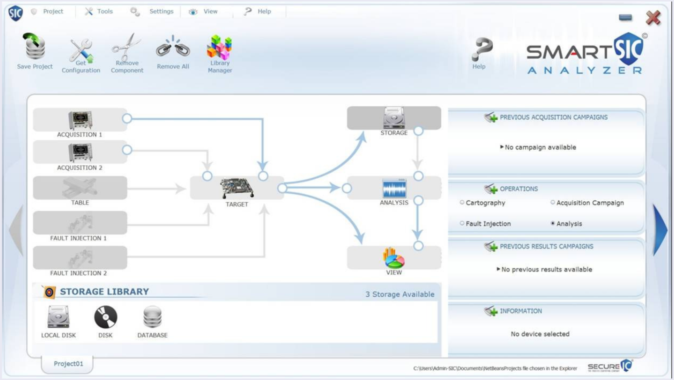
Security Evaluation of Countermeasures

Off-the-shelf platforms, *e.g.* the Smart-SIC Analyzer



Context

- + Security ☺ $\Longrightarrow$ + Costs ☹
- Trade-offs?
  - Maximal security within a given budget
  - Minimal spendings for a target security level (CC EALx?)
- Formal analysis: sound and realistic metrics for both security and cost.

Context

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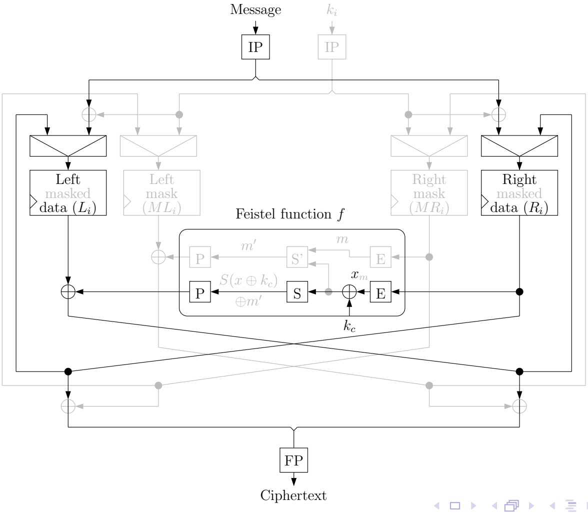
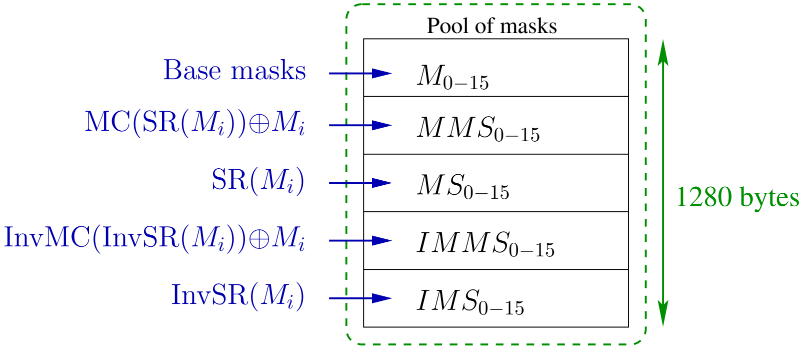
Masking with two (or more) paths



Masking with one path: $Z \to Z \oplus M$ (ex. AES)



- Homomorphic computation.
- This masking is the less costly in the litterature [NGDS12].
- Requires leak-free ROMs (well suited for ASIC & FPGA).

Performances

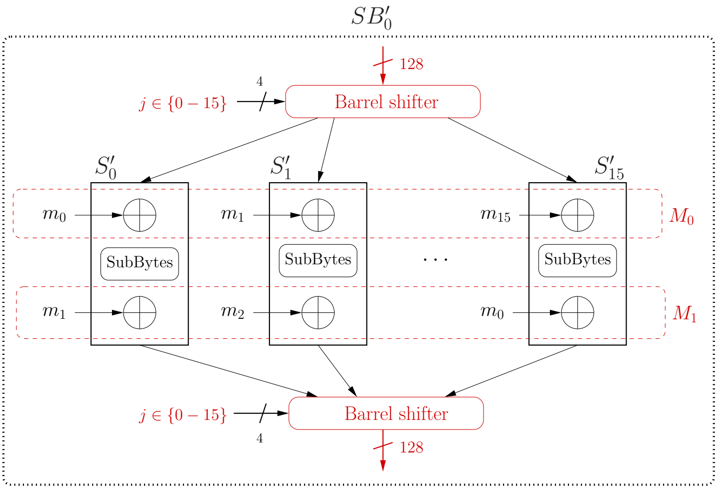
Table: Implementation results for reference and protected AES

	Unprotected	**RSM**	**Overhead**
Number of ALUTs (%)	2136 (8%)	2734 (10%)	28%
Number of M4K ROM Blocs (%)	20 (14%)	24 (17%)	20%
Frequency (MHz)	133	88	34%

Setting:

- $n = 8$ bit,
- 16 masks only, and (Price metric)
- provable security up to 2nd-order attacks (Security metric)

RSM mode of operation



RSM leakage

- Masked sboxes $Z \mapsto M_{\mathrm{out}} \oplus S(Z \oplus M_{\mathrm{in}})$.
- $$\mathcal{L}(Z, M) = \mathscr{L}(Z \oplus M) \quad .$$
  In this expression, Z and M are n-bit vectors, *i.e.* live in $\mathbb{F}_2^n$. The leakage function $\mathscr{L} : \mathbb{F}_2^n \to \mathbb{R}$ depends on the hardware.
  - In a conservative perspective, $\mathscr{L}$ is assumed to be bijective.
  - In a realistic perspective, $\mathscr{L}$ is assumed to non-injective.

Metrics

1. **Cost**: $\text{Card}[\mathcal{M}] \in \{1, \cdots, 2^n\}$.
2. **Security**:
   - Leakage: mutual information.
   - Attack: resistance against HO-CPA.

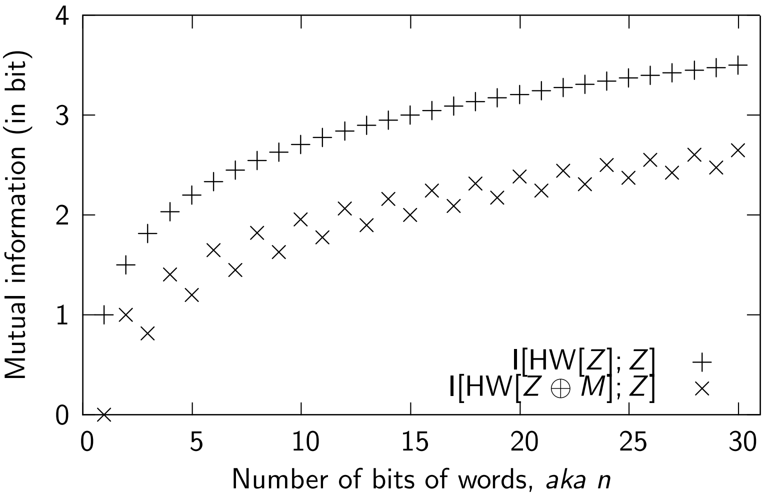
Modelization that bridges both notions:

$$\text{P}[M = m] = \begin{cases} 1/\text{Card}[\mathcal{M}] & \text{if } m \in \mathcal{M}, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

General Considerations

- $\forall \mathscr{L}, \mathrm{I}[\mathscr{L}(Z \oplus M); Z] = 0$ if $\mathrm{H}[M] = n$ bit (or equivalently, if $M \sim \mathcal{U}(\mathbb{F}_2^n)$). So with all the masks, the countermeasure is perfect.
- If $\mathscr{L}$ is bijective (*e.g.* $\mathscr{L} = \mathrm{Id}$), then $\mathrm{I}[\mathscr{L}(Z \oplus M); Z] = n - \mathrm{H}[M]$, irrespective of $\mathcal{M}$.
- If $\mathscr{L}$ is non-injective (*e.g.* $\mathscr{L} = \mathrm{HW}$), then $\mathrm{I}[\mathscr{L}(Z \oplus M); Z] < n - \mathrm{H}[M]$, but depends on $\mathcal{M}$.
  Motivating examples: for $\mathscr{L} = \mathrm{HW}$ on $n = 8$ bits,
  - $\mathrm{I}[\mathscr{L}(Z \oplus M); Z] = 1.42701$ bit if $\mathcal{M} = \{\mathtt{0x00}, \mathtt{0x0f}, \mathtt{0xf0}, \mathtt{0xff}\}$, but
  - $\mathrm{I}[\mathscr{L}(Z \oplus M); Z] = 0.73733$ bit if $\mathcal{M} = \{\mathtt{0x00}, \mathtt{0x01}, \mathtt{0xfe}, \mathtt{0xff}\}$.

Example for $\mathcal{M} = \{m, \neg m\}$



Optimal CPA

In [PRB09], it is explained that best possible dO-CPA has $\rho_{\mathrm{opt}}^{(d)}$:

$$\frac{\mathrm{Var}\left(f_{\mathrm{opt}}^{(d)}(Z)\right)}{\mathrm{Var}\left((\mathcal{L}(Z,M)-\mathrm{E}\mathcal{L}(Z,M))^d\right)} = \frac{\mathrm{Var}\left(\mathrm{E}\left(\left(\mathrm{HW}[Z\oplus M]-\frac{n}{2}\right)^d \mid Z\right)\right)}{\mathrm{Var}\left(\left(\mathrm{HW}[Z\oplus M]-\frac{n}{2}\right)^d\right)}$$

where

$$\begin{aligned} f_{\mathrm{opt}}^{(d)}(z) &\doteq \mathrm{E}\left((\mathcal{L}(Z,M)-\mathrm{E}\mathcal{L}(Z,M))^d \mid Z=z\right) \\ &= \frac{1}{\mathrm{Card}[\mathcal{M}]}\sum_{m\in\mathcal{M}}\left(\frac{-1}{2}\sum_{i=1}^{n}(-1)^{(z\oplus m)_i}\right)^d , \end{aligned}$$

noting that

$$\mathrm{E}\ \mathrm{HW}[Z\oplus M] = \frac{1}{\mathrm{Card}[\mathcal{M}]}\sum_{m\in\mathcal{M}}\frac{1}{2^n}\sum_{z\in\mathbb{F}_2^n}\mathrm{HW}[z\oplus m] = \frac{n}{2} .$$

Example for the intuition $(n = 4)$

	Card$[\mathcal{M}] = 2^4$	Card$[\mathcal{M}] = 2^3$	Card$[\mathcal{M}] = 2^2$	Card$[\mathcal{M}] = 2^1$
	0000	0000	0000	0000
	0001			
	0010			
	0011	0011	0011	
	0100	0100		
	0101			
	0110			
$\mathcal{M}$	0111	0111		
	1000	1000		
	1001			
	1010			
	1011	1011		
	1100	1100	1100	
	1101			
	1110			
	1111	1111	1111	1111

Example evaluation

Card$[\mathcal{M}]$	H$[M]$	$\rho_{\text{opt}}^{(1)}$	$\rho_{\text{opt}}^{(2)}$	I[HW$[Z \oplus M]; Z]$	I$[Z \oplus M; Z]$
2^4	4	0	0	0	0
2^3	3	0	0.166667	0.15564	1
2^2	2	0	0.333333	1.15564	2
2^1	1	0	1	1.40564	3
2^0	0	1	1	2.03064	4

- It seems that the most entropy, the least leakage in $\mathscr{L} = \text{HW}$ and in $\mathscr{L} = \text{Id}$.
- But this will be challenged by exhaustive searches...

Resistance against 1O-CPA and 2O-CPA

$$\rho_{\mathrm{opt}}^{(1)} = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{1}{\mathrm{Card}[\mathcal{M}]} \sum_{m \in \mathcal{M}} (-1)^{m_i} \right)^2 ,$$

$$\rho_{\mathrm{opt}}^{(2)} = \frac{1}{n(n-1)} \left(\frac{1}{\mathrm{Card}[\mathcal{M}]^2} \sum_{(m,m') \in \mathcal{M}^2} \left(\sum_{i=1}^{n} (-1)^{(m \oplus m')_i} \right)^2 - n \right) .$$

Expression in Boolean theory — With Indicator f of $\mathcal{M}$

- Boolean function $f : \mathbb{F}_2^n \to \mathbb{F}_2$, defined as:
  $\forall m \in \mathbb{F}_2^n, f(m) = 1 \iff m \in \mathcal{M}$.
- The Fourier transform $\hat{f} : \mathbb{F}_2^n \to \mathbb{Z}$ of the Boolean function $f : \mathbb{F}_2^n \to \mathbb{F}_2$ is defined as
  $\forall a \in \mathbb{F}_2^n, \hat{f}(a) \doteq \sum_{m \in \mathbb{F}_2^n} f(m)(-1)^{a \cdot m}$.
- It allows for instance to write
  $\mathrm{Card}[\mathcal{M}] = \sum_{m \in \mathcal{M}} 1 = \sum_{m \in \mathbb{F}_2^n} f(m) = \hat{f}(0)$. Recall $\mathrm{Card}[\mathcal{M}] \in [\![1, 2^n]\!]$, hence $\hat{f}(0) > 0$.

Expression of $\rho_{\mathrm{opt}}^{(1,2)}$ in Boolean theory

$$\rho_{\mathrm{opt}}^{(1)} = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{\hat{f}(e_i)}{\hat{f}(0)} \right)^2 , \qquad (1)$$

$$\rho_{\mathrm{opt}}^{(2)} = \frac{1}{n(n-1)} \sum_{\substack{(i,i') \in [\![1,n]\!]^2 \\ i \neq i'}} \left(\frac{\hat{f}(e_i \oplus e_{i'})}{\hat{f}(0)} \right)^2 . \qquad (2)$$

The e_i are the canonical basis vectors $(0, \cdots, 0, 1, 0, \cdots, 0)$.
Thus, RSM resists:

1. first-order attacks iff $\forall a$, $\mathrm{HW}[a] = 1 \Longrightarrow \hat{f}(a) = 0$;
2. first- and second-order attacks iff $\forall a$, $1 \leqslant \mathrm{HW}[a] \leqslant 2 \Rightarrow \hat{f}(a) = 0$.

Example: $n = 4$

All the functions $f : \mathbb{F}_2^4 \rightarrow \mathbb{F}_2$ that cancel $\rho_{\text{opt}}^{(1,2)}$.

f	HW[f]	H[M]	$\rho_{\text{opt}}^{(1,2)}$	I[HW[$Z \oplus M$]; Z]	I[$Z \oplus M$; Z]	$d_{\text{alg}}^{\circ}(f)$
0x3cc3	8	3	0,0	0.219361	1	1
0x5aa5	8	3	0,0	0.219361	1	1
0x6699	8	3	0,0	0.219361	1	1
0x6969	8	3	0,0	0.219361	1	1
0x6996	8	3	0,0	1	1	1
0x9669	8	3	0,0	1	1	1
0x9696	8	3	0,0	0.219361	1	1
0x9966	8	3	0,0	0.219361	1	1
0xa55a	8	3	0,0	0.219361	1	1
0xc33c	8	3	0,0	0.219361	1	1
0xffff	16	4	0,0	0	0	0

Functions f are classified by equivalence relationships

- Let us call σ a permutation of $[\![1, n]\!]$. Thus $\rho_{\mathrm{opt}}^{(1,2)}(f \circ \sigma) = \rho_{\mathrm{opt}}^{(1,2)}(f)$.
- The complementation $\rho_{\mathrm{opt}}^{(1,2)}(\neg f) = \rho_{\mathrm{opt}}^{(1,2)}(f)$.

Solutions are derived from: $f(x_1, x_2, x_3, x_4) = x_1 \oplus x_2 \oplus x_3, \bigoplus_i x_i, 1$.
Note: $\mathcal{M}$ does not decompose as $\tilde{\mathcal{M}} \cup \neg\tilde{\mathcal{M}}$,

Case $n = 5$

Nb. classes	HW[f]	H[M]	$\rho_{\text{opt}}^{(1)}$	$\rho_{\text{opt}}^{(2)}$	I[HW[$Z \oplus M$]; Z]	I[$Z \oplus M$; Z]	$d_{\text{alg}}^{\circ}(f)$
3	**8**	**3**	**0**	**0**	**0.32319**	**2**	**2**
4	**12**	**3.58496**	**0**	**0**	**0.18595**	**1.41504**	**3**
2	**16**	**4**	**0**	**0**	**0.08973**	**1**	**1**
2	**16**	**4**	**0**	**0**	**0.08973**	**1**	**2**
4	16	4	0	0	0.12864	1	2
2	16	4	0	0	0.16755	1	1
4	16	4	0	0	0.26855	1	2
6	16	4	0	0	0.32495	1	2
1	16	4	0	0	1	1	1
4	**20**	**4.32193**	**0**	**0**	**0.07349**	**0.67807**	**3**
3	**24**	**4.58496**	**0**	**0**	**0.04300**	**0.41504**	**2**
1	**32**	**5**	**0**	**0**	**0**	**0**	**0**

Here, we start to see the compromize, with good choices in **bold**.

SAT solvers

- f is a 2^n Boolean variable set, noted $\{f_x = f(x), x \in \mathbb{F}_2^n\}$.
- For every value Price (defined as $\text{Card}[\mathcal{M}]$), we have:

$$\forall a, 1 \leqslant \text{HW}[a] \leqslant 2, \quad \sum_x f(x)(-1)^{a \cdot x} = 0 \quad \Longleftrightarrow$$

$$\forall a, 1 \leqslant \text{HW}[a] \leqslant 2, \sum_x f_x \wedge (a \cdot x) = \frac{\sum_x f_x}{2} = \frac{\text{Card}[\mathcal{M}]}{2} \quad .$$

- More precisely, any condition "$\leqslant k(f_1, \cdots, f_n)$", for $0 \leqslant k \leqslant n$, can be expressed in terms of CNF clauses [Sin05]. We note that:

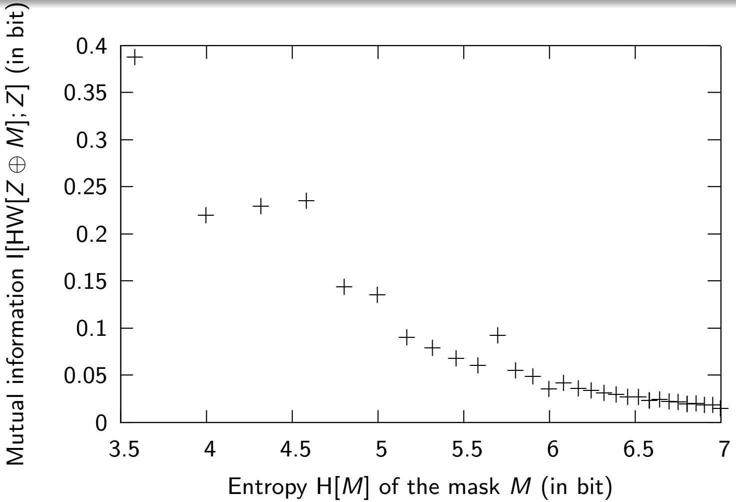
$$\text{HW}[f] \leqslant k \quad \Leftrightarrow \quad n - \text{HW}[\neg f] \leqslant k \quad \Leftrightarrow \quad \text{HW}[\neg f] \geqslant n - k \, .$$

- Example: 256 literals, but 1,105,664 auxiliary variables and 2,219,646 clauses, irrespective of $\text{Card}[\mathcal{M}] \in \mathbb{N}^*$.

Summary for $n = 8$

- $\mathrm{Card}[\mathcal{M}] = 12$. One MIA found, 0.387582 bit
- $\mathrm{Card}[\mathcal{M}] = 16$. Many MIA, in $[0.181675, 1.074950]$ bit.
- There are solutions only for $\mathrm{Card}[\mathcal{M}] \in \{4 \times \kappa, \kappa \in [\![3, 61]\!] \cup \{64\}\}$.

Example of solutions



Conclusions

- It is possible to achieve high-order security even with depleted entropy
- Case treated in the presentation: Resist 1O-CPA and 2O-CPA, with fewer masks as possible.
- We discovered that Card$[\mathcal{M}]$ was not the only variable $\Rightarrow$ solutions actually depend on $\mathcal{M}$.
- An encoding in terms of indicator function f of $\mathcal{M}$ shows that we are looking for 2nd order correlation-immune Boolean functions of lowest weight.
- Secure even if $\mathcal{M}$ is public.

Perspectives

- Find other functions for $n > 8$.
- Algebraic constructions:
  - Maiorana-McFarland, or
  - codes of dual-distance d...
- Dynamic reconfiguration to update $\mathcal{M}$ on a regular basis?

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