Mars Attacks! Revisited.

Differential Attack 12 Rounds of the MARS Core and Defeating the Complex MARS Key Schedule

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Motivation

What is MARS?

- block cipher with 128 bit block size
- developed 1998 by a team from IBM as a candidate for the Advanced Encryption Standard (AES)
- one of five finalists in the AES competition 2001
- no attacks from 2001 till 2009

Motivation (cont'd)

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- full AES is theoretically broken
- many attacks on AES base on exploiting the relatively weak key schedule of AES
- MARS structure differs from other ciphers (mixing rounds)
- key scheduler much stronger/ more complex than key scheduler of AES

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We propose two attacks:

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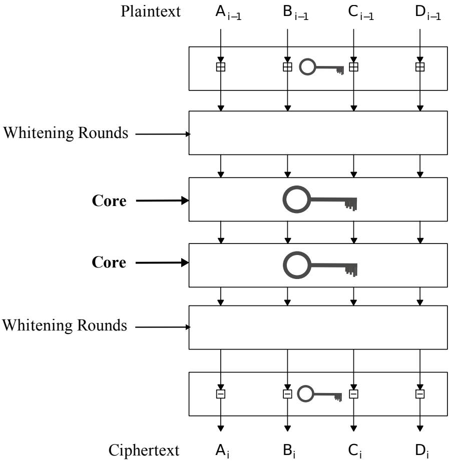
- extend 11-round distinguisher by Kelsey et al to 12 core rounds
- establish first key recovery attack on the MARS key schedule, using the distinguisher to recover the secret key

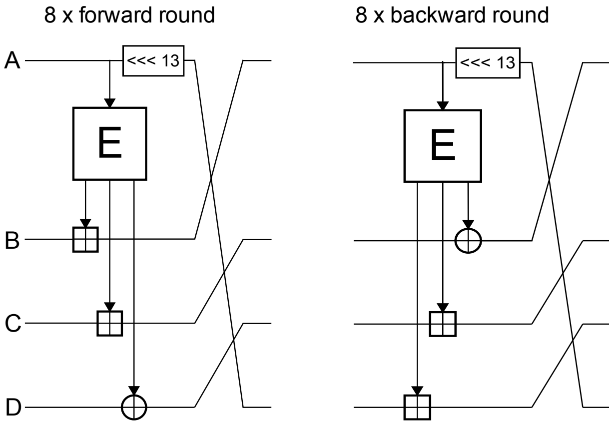
Outline

MARS



- 128 bit block size
- internal state: 4×32 bit words (A, B, C, D)

MARS - Structure of the Core Rounds



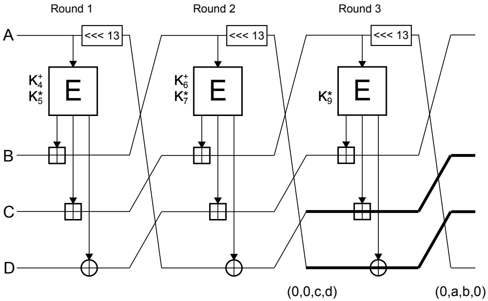
Distinguisher and Subkey Recovery

Exploits differential properties of the MARS core

- 3-round differential characteristic with probability 1
  $(0, 0, 0, \alpha) \rightarrow (\beta, 0, 0, 0)$
- distinguisher uses the 3-rounds characteristic twice, for rounds 4 - 6 and 7 - 9
- differences, if multiplied with a constant, propagate only in the most significant bits (used in round 10)

Distinguisher and Subkey Recovery (cont'd)

For each of the 2^{154} subkey candidates of the first three rounds do:



1. choose 2^{56} texts with arbitrary differences $(0, a, b, 0)$

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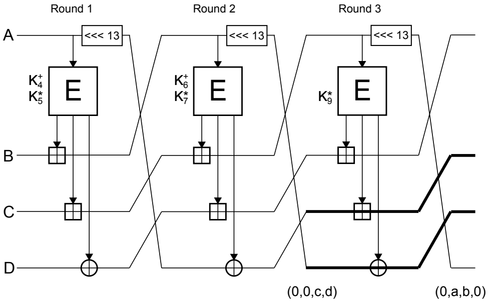

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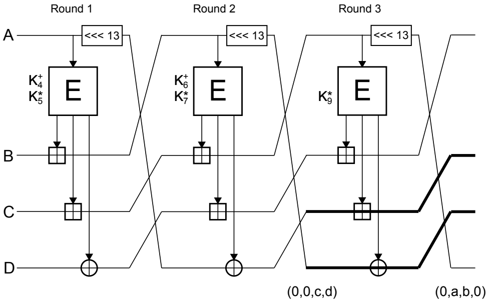

1. choose 2^{56} texts with arbitrary differences $(0, a, b, 0)$
2. partially decrypt $(0, a, b, 0)$ to reach (A, B, C, D)
3. create 2^{56} batches with 302 texts each with difference (A, B, C, D) between batches

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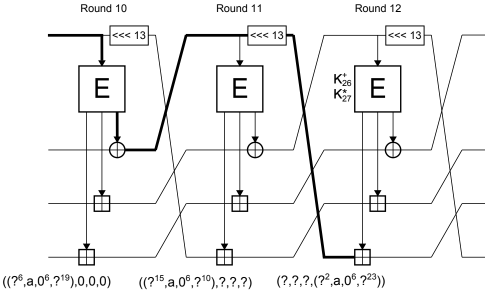

5. partially decrypt all ciphertexts with each of the 2^{32} subkey candidates for Round 12 and extract the bit "*a*" for each ciphertext

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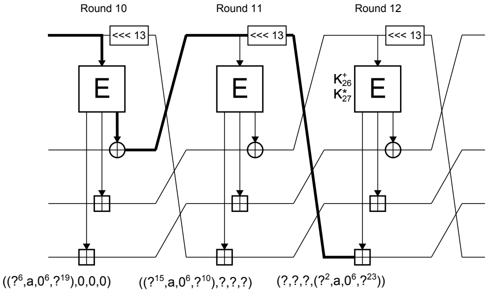

5. partially decrypt all ciphertexts with each of the 2^{32} subkey candidates for Round 12 and extract the bit "*a*" for each ciphertext
6. build 2^{56} strings of 302 "*a*" bits for each batch

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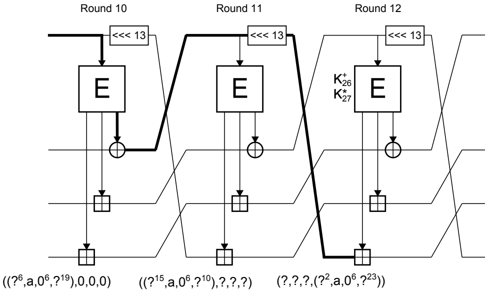

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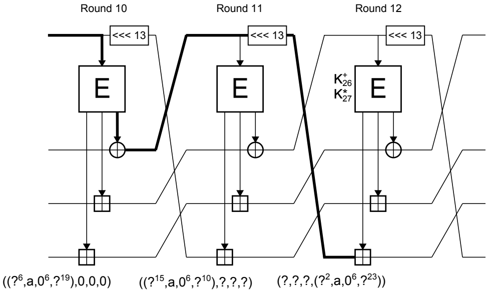

7. store and sort the resulting bit strings in order of the chosen plaintexts
8. compare the bit strings pairwise to identify the correct subkey candidate

What we got from the Distinguisher

valid subkeys for

$$\{K_4^+, K_5^*, K_6^+, K_7^*, K_9^*, K_{26}^+, K_{27}^*(9 \text{ bit})\}.$$

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- uses internal array $T[0 \ldots 14]$ with 15×32-bit words
- three phases per iteration:
  - linear transformation
  - four stirring rounds
  - removing patterns from multiplication keys

Key Schedule (cont'd)

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- **Four stirring rounds**
  for ($k = 1, \ldots, 4$)
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- **Storing next 10 keys**
  for $(i = 0, \dots, 9)$
  $K[10j + i] = T[4i \bmod 15]$

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- idea: mount a Meet-in-the-Middle-Attack on the first iteration

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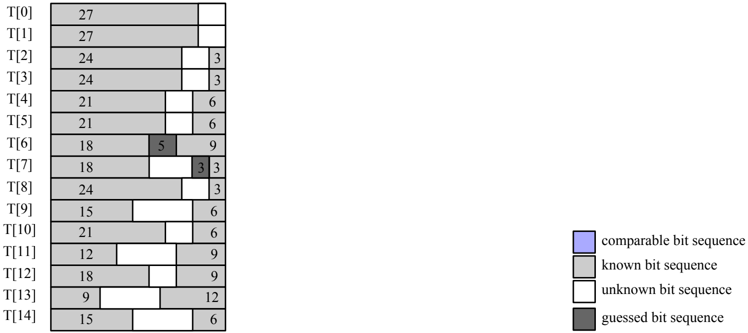

- Linear Transformation:

$$T[i] = T[i] \oplus ((T[i - 7 \bmod 15] \oplus T[i - 2 \bmod 15]) \lll 3) \oplus (4i + j)$$

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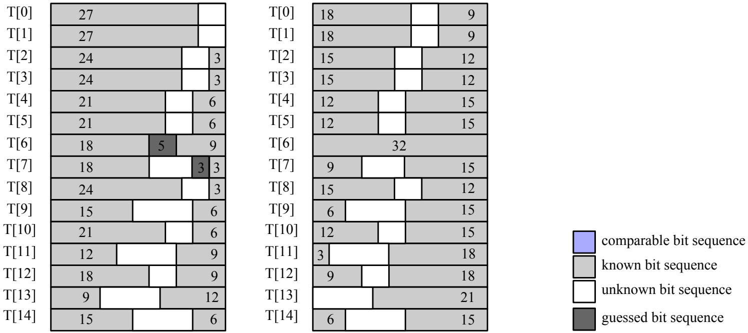

- First Stirring Round:

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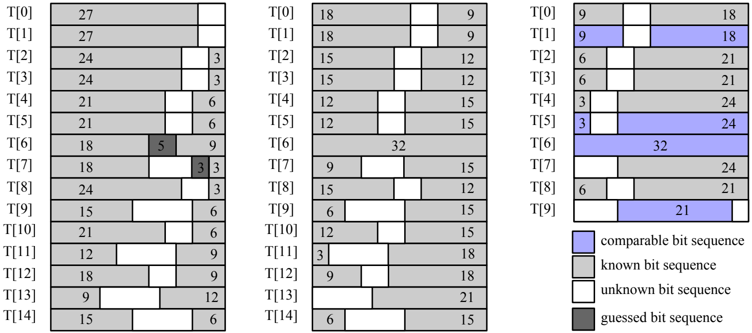

- Second Stirring Round:

  $T[i] = (T[i] + S[\text{low 9 bits of } T[i-1 \bmod 15]]) \lll 9$

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- our distinguisher recovers five subkeys from first iteration: $\{K_4^+, K_5^*, K_6^+, K_7^*, K_9^*\}$

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- our distinguisher recovers five subkeys from first iteration: $\{K_4^+, K_5^*, K_6^+, K_7^*, K_9^*\}$
- attack uses four subkeys that are mapped to $T[i]$s as follows: $\{K_4^+, K_5^*, K_6^+, K_9^*\} \rightarrow \{T[1], T[5], T[9], T[6]\}$

MITM - Backward Step

- **Modification of multiplication keys**
  - invert multiplication keys $\{K_5^*, K_9^*\}$
  - lookup table for K $\rightarrow$ T projections
  - max. $102 \approx 2^7$ candidates
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  - invert multiplication keys $\{K_5^*, K_9^*\}$
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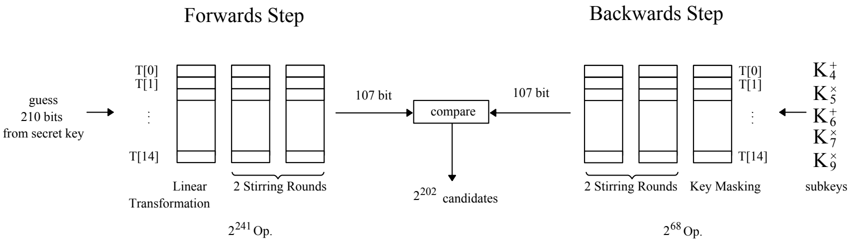
- **Two stirring rounds backwards**
  - require least significant nine bits for each of our four words $T[i]$ for each stirring round
  - know the bits for $T[6]$ after guessing $T[5]$
  - $2^{9 \cdot 3 \cdot 2}$ op. $= 2^{54}$ op.
- $2^{14} \cdot 2^{54}$ op. $= 2^{68}$ op. for backward step

MITM Attack on the MARS Key Schedule



Attack Analysis

Distinguisher operations:

- 2^{65} Texts $\cdot 2^{186}$ Keys $\cdot 3$ Executions $\approx 2^{252}$ *Encryptions*
- 3 executions are required as one 3-round differential for round 7-9 has probability $\neq 1$

Attack Analysis

Forward step:

- guessing the bits of $T[0] \ldots T[7]$: 2^{210}
- guessing 5 bit of $T[6]$ and 3 bit of $T[7]$: 2^{8}
- carry bit for 23 additions: 2^{23}
- **summarize:** 2^{241}

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Backward step:

- nine bits for $T[0]$, $T[4]$, $T[8]$ (two stirring rounds): 2^{54}
- multiplication keys (from possible table entries): 2^{14}
- **summarize:** 2^{68}

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- We gather 2^{202} candidates for 210 bits of the secret key
- $2^{202} \cdot 2^{46} = 2^{248}$ Op. for final testing

Conclusion

- we have ...
  - extended the 11-round attack by Kelsey et al to a differential attack on 12 rounds
  - suggested a MITM attack on the MARS key schedule that allows to recover the secret key more efficiently than exhaustive search

Recent Attacks on MARS/Analysis

Type	Rounds	Texts	Bytes	Op.	Reference
Differential	**12C**	2^{65}	2^{69}	2^{252}	**this work**
Amp. Boomerang	11C	2^{65}	2^{70}	2^{229}	[KKS00]
Amp. Boomerang	6M, 6C	2^{69}	2^{73}	2^{197}	[KS00]
MITM	16M, 5C	8	2^{236}	2^{232}	[KS00]
Diff. MITM	16M, 5C	2^{50}	2^{197}	2^{247}	[KS00]
Impossible Diff.	8C	-	-	-	[BF00]
Differential	8M, 8C	2^{105}	2^{109}	2^{231}	[Pes09]

Table: Op: operations, C: core rounds, M: mixing rounds